Answers mid-term exam
Quantum Physics 1   24 Sept. 2015

1

$\boxed{T1}$ a) As a starting point you can use that for a normalized state $|c_1|^2 + |c_2|^2 = 1$.
For the state as given, $|c_1|^2 + |c_2|^2 = |\sqrt{5}|^2 + |e^{i\varphi}\sqrt{10}|^2 = 15$
This state becomes normalized (while representing the same physical state) when describing it as

$$|\Psi_{0,\,NORM}\rangle = \frac{c_1}{\sqrt{|c_1|^2+|c_2|^2}}|\varphi_1\rangle + \frac{c_2}{\sqrt{|c_1|^2+|c_2|^2}}|\varphi_2\rangle = \sqrt{\frac{5}{15}}|\varphi_1\rangle + e^{i\varphi}\sqrt{\frac{10}{15}}|\varphi_2\rangle$$

b) Use $\hat{U} = e^{-\frac{i}{\hbar}\hat{H}t}$ and $e^{i\alpha} = e^{i\pi} = -1$

$$|\psi(t)\rangle = \hat{U}|\psi(t{=}0)\rangle = \hat{U}|\Psi_2\rangle = e^{-\frac{i}{\hbar}E_1 t}\cdot -1 \cdot \sqrt{\tfrac{1}{2}}|\varphi_1\rangle + \frac{e^{-\frac{i}{\hbar}E_2 t}}{\sqrt{2}}|\varphi_2\rangle$$

c) $\langle \hat{A}\rangle(t) = \langle \psi(t)|\hat{A}|\psi(t)\rangle =$ { Use $\omega_1 = \frac{E_1}{\hbar}$, $\omega_2 = \frac{E_2}{\hbar}$ and see also b)

$$\frac{1}{2}\left(-e^{i\omega_1 t}\langle\varphi_1| + e^{i\omega_2 t}\langle\varphi_2|\right)\hat{A}\left(-e^{-i\omega_1 t}|\varphi_1\rangle + e^{-i\omega_2 t}|\varphi_2\rangle\right) =$$

$$\frac{1}{2}\left(-e^{-i(\omega_2-\omega_1)t}A_0 - e^{+i(\omega_2-\omega_1)t}A_0\right)$$

{ Using $\langle\varphi_1|\hat{A}|\varphi_1\rangle = 0$, $\langle\varphi_2|\hat{A}|\varphi_2\rangle = 0$, $\langle\varphi_1|\hat{A}|\varphi_2\rangle = \langle\varphi_2|\hat{A}|\varphi_1\rangle = A_0$

$= -A_0 \cos((\omega_2 - \omega_1)t)$ and since $\omega_2 = 2\omega_1$

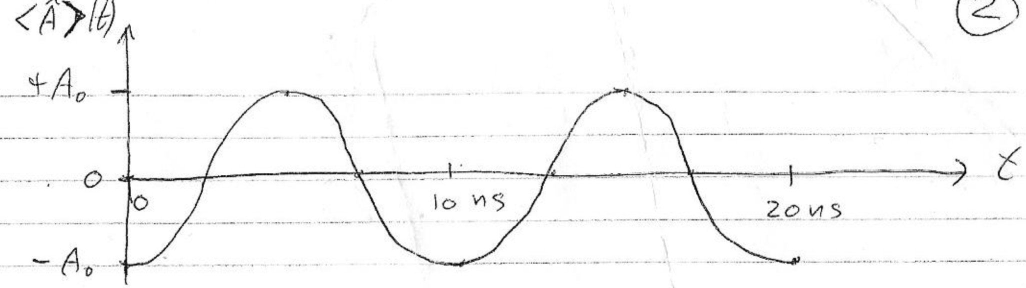
$$\langle\hat{A}\rangle(t) = -A_0\cos(\omega_1 t)$$

$A_0 = 1\ e\ nm$ → is the amplitude of osc.

$$\omega_1 = 2\pi f_1 = 2\pi\frac{E_1}{h} = 2\pi\frac{6.626\cdot 10^{-26}}{6.626\cdot 10^{-34}}\ Hz = 2\pi\cdot 10^{8}\ Hz$$

⇒ period of oscillation is $T = \frac{1}{f_1} = 10$ ns

2



$\boxed{T2}$ a) The answer is $\Psi(x)$, where $\Psi(x)$ is the Fourier transform of $\bar{\Phi}(k)$ ⇒

$$\Psi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ikx}\,\bar{\Phi}(k)\,dk = \frac{1}{\sqrt{2\pi a}}\int_{-a/2}^{a/2} e^{+ikx}\,dk$$

$$= \frac{1}{\sqrt{2\pi a}}\left[\frac{1}{ix}e^{ikx}\right]_{-a/2}^{a/2} = \frac{1}{\sqrt{2\pi a}}\,\frac{1}{x}\,2\sin\left(\frac{ax}{2}\right)$$

⟶ write in sinc form ⇒ $\Psi(x) = \sqrt{\frac{a}{2\pi}}\,\frac{\sin(\frac{a}{2}x)}{(\frac{a}{2}x)}$

(This is a real and even function of x.)

b) $\langle x\rangle = \int_{-\infty}^{\infty}\bar{\Phi}(k)^*\,\hat{x}\,\bar{\Phi}(k)\,dk = \int_{-\infty}^{\infty}\Psi(x)^*\,\hat{x}\,\Psi(x)\,dx$
$= \int_{-\infty}^{\infty}\Psi(x)^*\,x\,\Psi(x)\,dx$.

Since $\Psi(x)$ is real and even, this gives $\langle x\rangle = 0$

c) $p_x = \hbar k$. In the superposition described by $\bar{\Phi}(k)$, the highest value of k in the state is $+a/2$. ⇒

The maximum p_x that can be a measurement outcome is $p_x = \hbar\cdot\frac{a}{2}$ ⇒

$$p_x = 1.055\cdot 10^{-34}\ Js\cdot 1\cdot 10^{10}\ m^{-1} = 1.055\cdot 10^{-24}\ \overline{kg\ m/s}$$

(unit of p_x